Presentation follows Mueller 2003

May's Theorem: The only group decision function that satisfies decisiveness, anonymity, neutrality and positive responsiveness is simple majority rule.

Definitions: *group decision function:* $D = f(D_1, D_2, , D_n)$, n is the number of individuals in the group. Each D_i takes on the value 1, 0 or -1 and corresponds to the preferences of the individuals. Hence, 1 means that individual i strictly prefers x to y, -1 means that individual i striclty prefers y to x and 0 means that individual i is indifferent between x and y. Thus, each D_i corresponds to a ballot an individual i casts between two alternatives. $F(.)$ represents an aggregation rule that determines the winning issue.

Simple majority rule can thus be defined in the following way:

$$(\sum_{i=1}^{n} D_i > 0) \rightarrow D = 1$$
$$(\sum_{i=1}^{n} D_i = 0) \rightarrow D = 0$$
$$(\sum_{i=1}^{n} D_i < 0) \rightarrow D = -1$$

In words, this definition means that x is prefered to y by the group, if the number of individuals strictly prefering x to y is higher than the number of individiduals strictly prefering y to x. Some authors prefer to call this aggregation rule plurality rule! Simple majority rule is then defined as: the group strictly prefers x to y iff the number of individuals strictly prefering x to y is higher than $\frac{n}{2}$.

- *Decisiveness*: For all preference profiles the group decision function is defined and single-valued.
- *Anonymity*: D is determined only by the values of D_i, and is independent of how they are assigned. Any permutation of these ballots leaves D unchanged.
- *Neutrality*: If x defeats (ties) y for one set of individual preferences, and all individuals have the same ordinal rankings for z and w as for x and y then z defeats (ties) w.
- *Positive responsiveness*: If D equals 0 or 1, and one individual changes his preferences from -1 to 0 or 1, or from 0 to 1, and all other individual preference relations remain unchanged, then $D = 1$. Some authors prefer to call positive responsiveness monotonicity!

References
Mueller, D. (2003): Public Choice III, Cambridge: Cambridge University Press
Austen-Smith, D./Banks, J. (1999): Positive Political Theory I, Princeton: Princeton University Press